

Find any points of discontinuity for each rational function.

1) $y = \frac{2x^2 + 5}{x^2 - 2x} = \frac{2x^2 + 5}{x(x-2)}$

VA: $x = 0$
 $x = 2$

2) $y = \frac{x^2 + 2x}{x^2 + 2} = \frac{x(x+2)}{x^2 + 2}$

No
discontinuities
($x^2 + 2 \neq 0$)

3) $y = \frac{3x - 3}{x^2 - 1} = \frac{3(x-1)}{(x+1)(x-1)}$

Hole at $(1, \frac{3}{2})$

VA: $x = -1$

6) $y = \frac{x^2 + 4x + 3}{2x^2 + 5x - 7} = \frac{(x+3)(x+1)}{(2x+7)(x-1)}$

VA: $x = -\frac{7}{2}$
 $x = 1$

7) $y = \frac{x^3 - 8}{x^3 - 8} = 1$

Hole at $(2, 1)$

8) $y = \frac{1}{2x^2 + 3x - 7}$

VA: $x = \frac{-3 + \sqrt{65}}{4}$

$x = \frac{-3 - \sqrt{65}}{4}$

13) $y = \frac{(x+3)(x-2)}{(x-2)(x+1)}$

Hole at $(2, \frac{5}{3})$

VA: $x = -1$

15) $y = \frac{x+5}{x^2+9}$

no
discontinuities

$(x^2 + 9 \neq 0)$

16) $y = \frac{9 - x^2}{x^2 - 9} = \frac{(3+x)(3-x)}{(x+3)(x-3)}$

$= -1$

Holes at: $(-3, -1)$ and
 $(3, -1)$

17) $y = \frac{2x^2}{2x^2 + 2}$

No discontinuities

$(2x^2 + 2 \neq 0)$

18) $y = \frac{6x^2 + x - 2}{3x^2 + 17x + 10}$

$\frac{(3x+2)(2x-1)}{(3x+2)(x+5)}$

Hole at $(-\frac{2}{3}, -\frac{7}{13})$

VA: $x = -5$

A. $y = \frac{(x-2)^3}{(x-2)^2} = x - 2$

Hole at $(2, 0)$ End Behavior

Find the horizontal asymptote of the graph of each rational function.

19) $y = \frac{5}{x+6}$

$y = 0$

20) $y = \frac{x+2}{2x^2-4}$

$y = 0$

B. $y = \frac{x^3 + 2}{x^2 - 4}$

EBA:
 $y = x$

(Long -
Division)

$$21) y = \frac{x+1}{x+5}$$

$$y = 1$$

$$22) y = \frac{x^2+2}{2x^2-1}$$

$$y = \frac{1}{2}$$

$$c. y = \frac{3x^2+4x-1}{x-1}$$

$$\begin{array}{r} 3 \quad 4 \quad -1 \\ \downarrow \quad \downarrow \quad \downarrow \\ 3 \quad 7 \quad 6 \end{array}$$

$$\text{EBA: } y = 3x + 7$$

$$23) y = \frac{5x^3+2x}{2x^5-4x^3}$$

$$y = 0$$

$$d. y = \frac{x^3+5x}{x+3}$$

$$\begin{array}{r} -3 \mid 1 \quad 0 \quad 5 \quad 0 \\ \downarrow \quad \downarrow \quad \downarrow \\ 1 \quad -3 \quad 14 \end{array}$$

$$\text{EBA: } x^2 - 3x + 14 = y$$

$$24) y = \frac{3x-4}{4x+1}$$

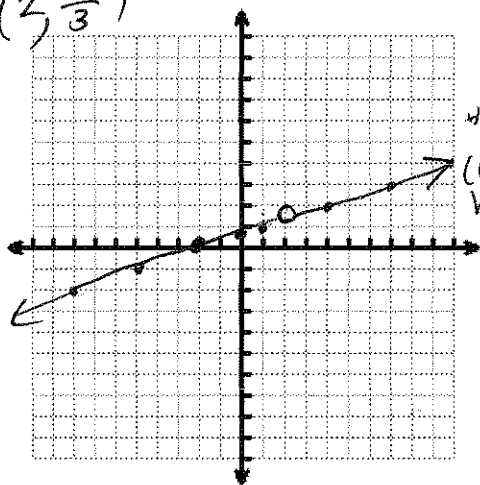
$$y = \frac{3}{4}$$

Sketch the graph of each rational function.

$$25) y = \frac{x^2-4}{3x-6} = \frac{(x+2)(x-2)}{3(x-2)} = \frac{1}{3}(x+2)$$

$$\text{Hole: } (2, \frac{4}{3})$$

y-int: $(0, \frac{2}{3})$
x-ints: $(-2, 0)$
no VA

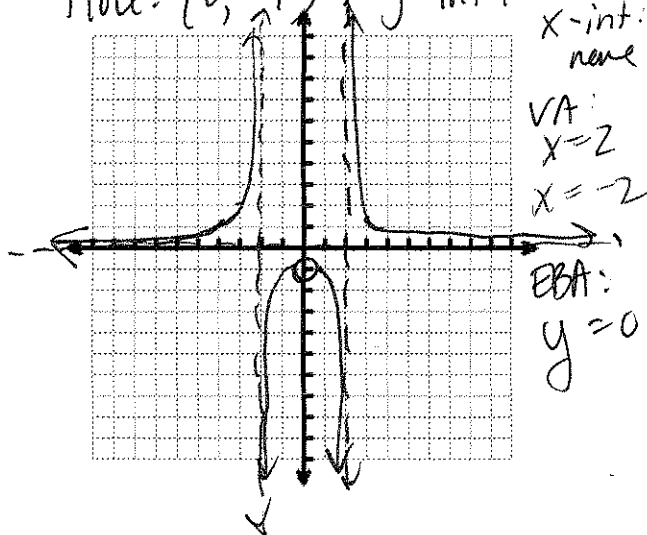


$= \frac{1}{3}(x+2)$
 $= \frac{1}{3}x + \frac{2}{3}$
a line!
(with a hole in it)

$$26) y = \frac{4x}{x^3-4x} = \frac{4x}{x(x^2-4)} = \frac{4x}{x(x+2)(x-2)}$$

$$\text{Hole: } (0, -1)$$

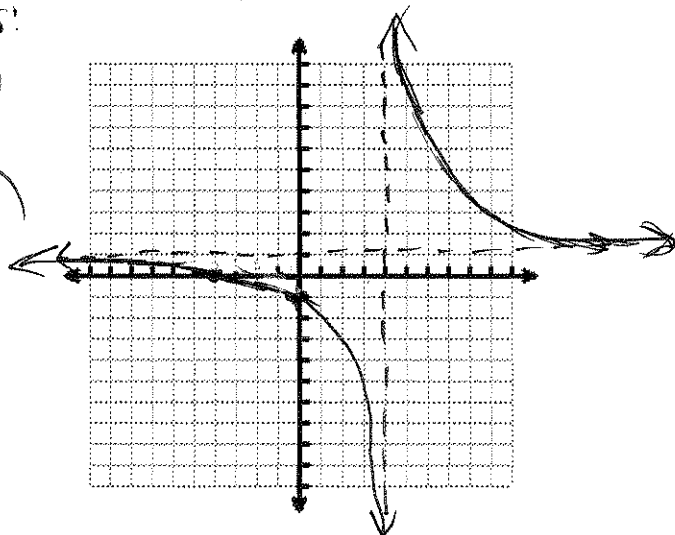
y-int: none
x-int: none
VA: $x=2$
 $x=-2$
EBA: $y=0$



$$27) y = \frac{x+4}{x-4}$$

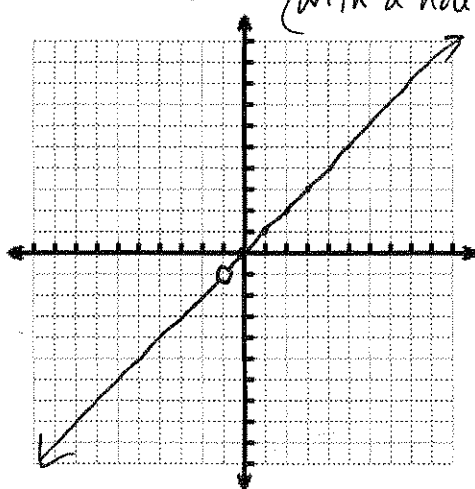
no holes
VA: $x=4$

x-ints: $(-4, 0)$
y-int: $(0, -1)$
EBA: $y=1$

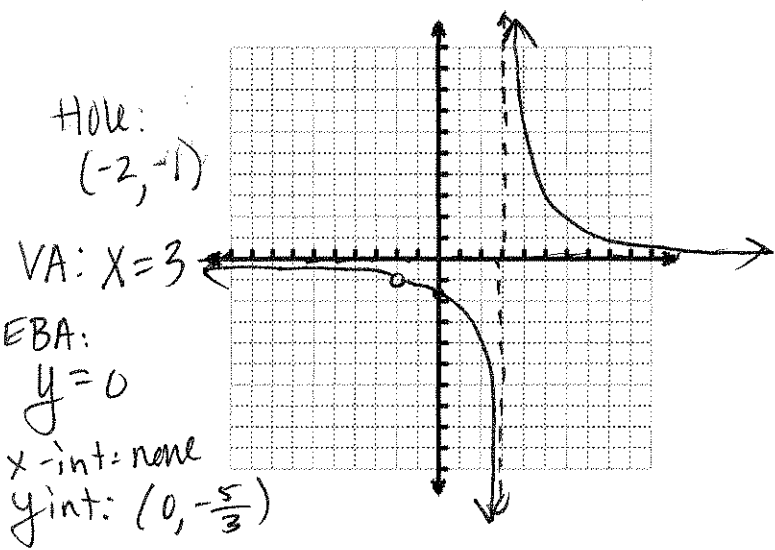


$$28) y = \frac{x(x+1)}{x+1} = x$$

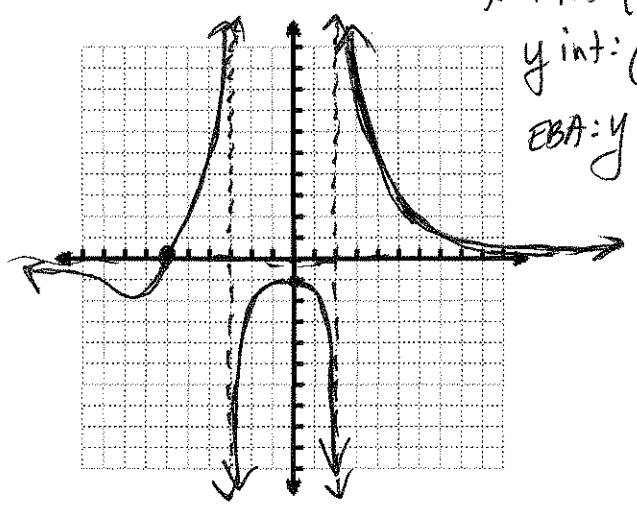
a line!
Hole: $(-1, -1)$
(with a hole in it)



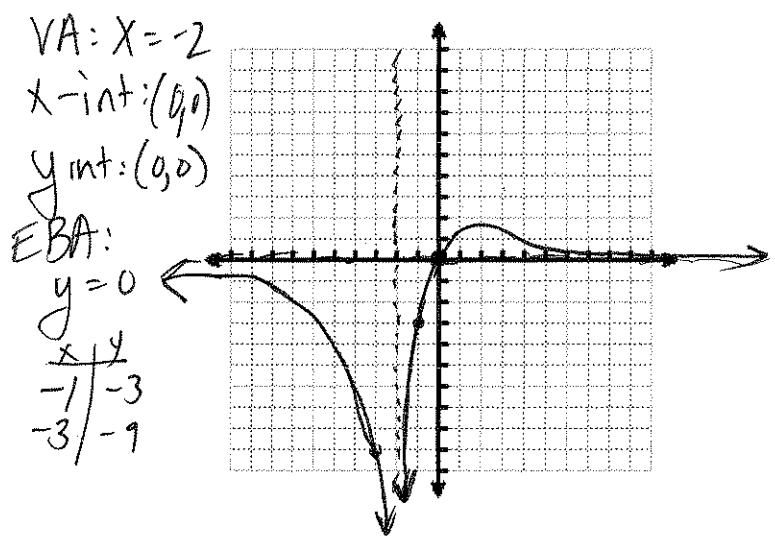
E. $y = \frac{5x+10}{x^2-x-6} = \frac{5(x+2)}{(x-3)(x+2)}$



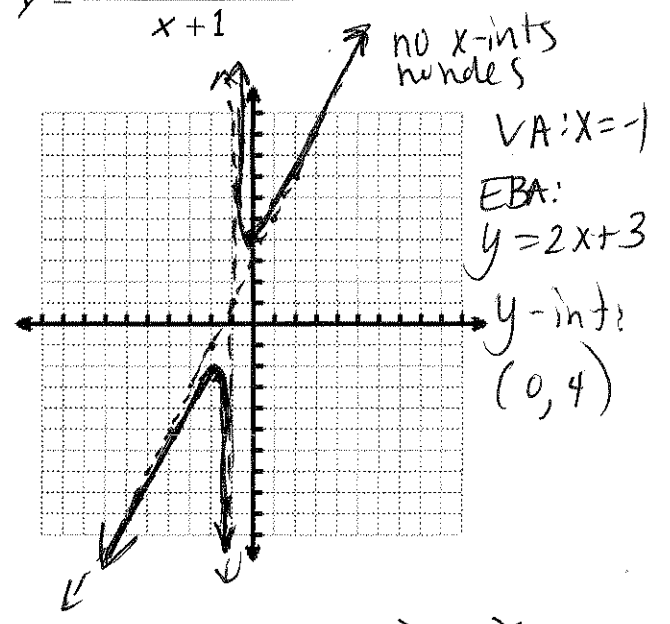
29) $y = \frac{x+6}{(x-2)(x+3)}$ no holes
 VA: $x=2$ $x=-3$
 x-ints: $(-6, 0)$
 y-int: $(0, -1)$
 EBA: $y=0$



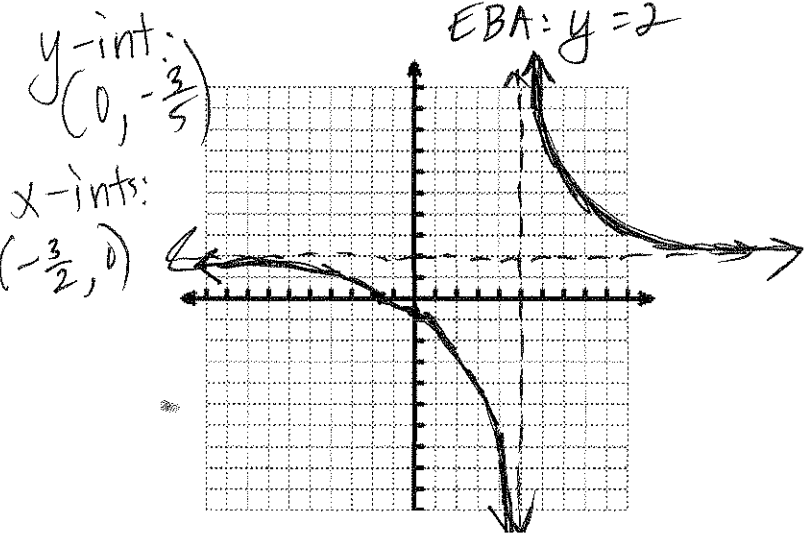
30) $y = \frac{3x}{(x+2)^2}$



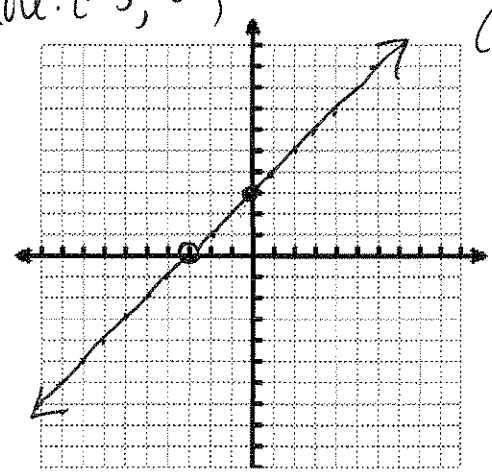
F. $y = \frac{2x^2+5x+4}{x+1}$ ← has no real zeros



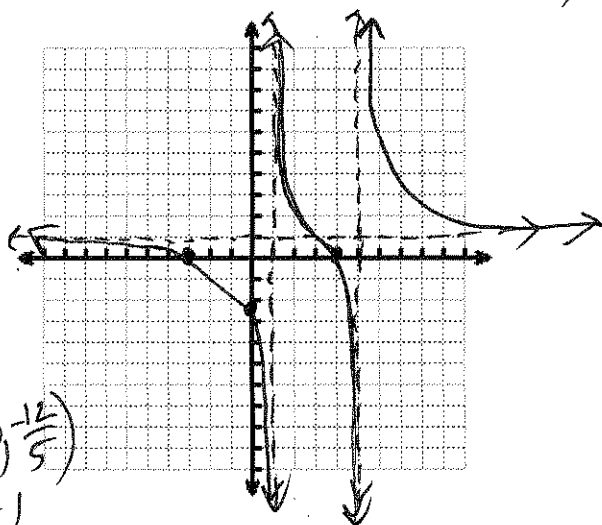
35) $y = \frac{2x+3}{x-5}$ VA: $x=5$
 EBA: $y=2$



36) $y = \frac{x^2+6x+9}{x+3} = \frac{(x+3)(x+3)}{(x+3)} = x+3$
 Hole: $(-3, 0)$
 a line! (with a hole in it)



$$37) y = \frac{x^2 - x - 12}{x^2 - 6x + 5} = \frac{(x-4)(x+3)}{(x-5)(x-1)}$$



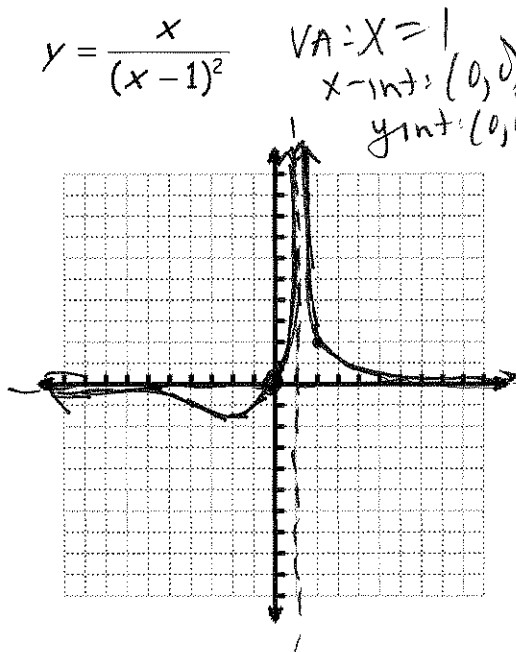
VA:
 $x=5$
 $x=1$

x-ints:
 $(4, 0)$
 $(-3, 0)$

y-int: $(0, \frac{12}{5})$

EBA: $y=1$

$$38) y = \frac{x}{(x-1)^2}$$

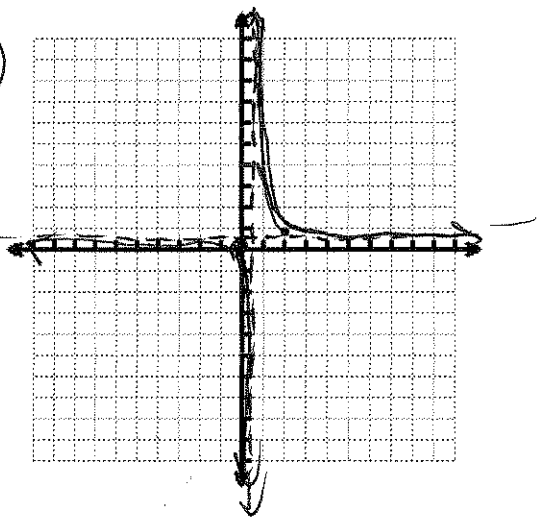


VA: $x=1$
 x-int: $(0, 0)$
 y-int: $(0, 0)$

EBA:
 $y=0$

x	y
$\frac{x}{2}$	$\frac{y}{2}$

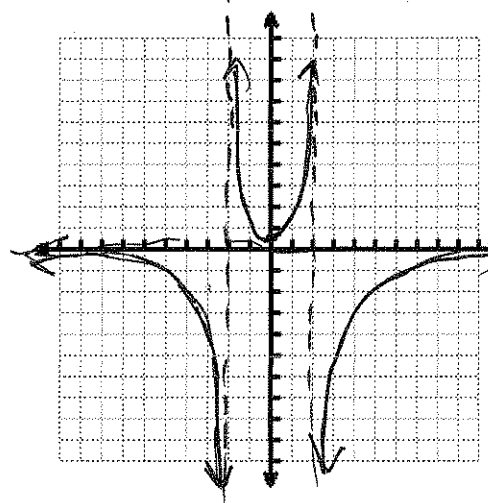
$$39) y = \frac{2x}{3x-1}$$



VA: $x = \frac{1}{3}$
 x-int: $(0, 0)$
 y-int: $(0, 0)$

EBA:
 $y = \frac{2}{3}$

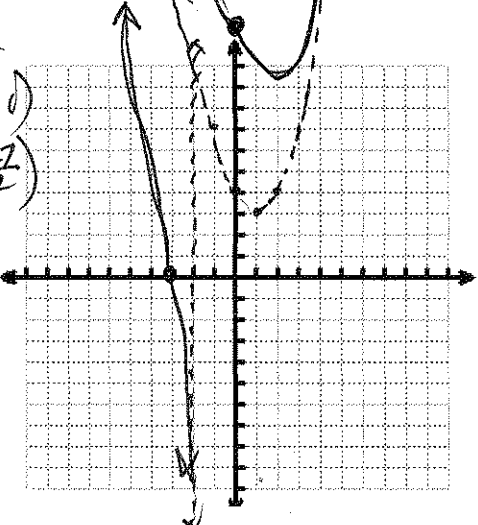
$$40) y = \frac{-2}{x^2 - 4} = \frac{-2}{(x+2)(x-2)}$$



VA: $x = -2$
 $x = 2$
 y-int: $(0, \frac{1}{2})$

no x-ints
 EBA:
 $y = 0$

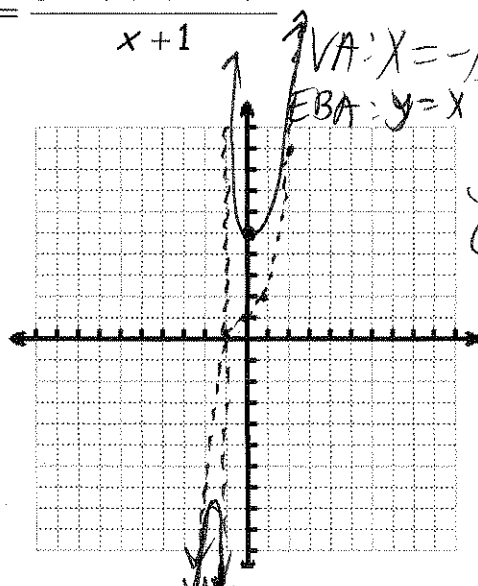
$$G. y = \frac{x^3 + 27}{x+2} = \frac{(x+3)(x^2 - 3x + 9)}{x+2}$$



VA: $x = -2$
 x-int: $(-3, 0)$
 y-int: $(0, \frac{27}{2})$

EBA:
 $y = x^2 - 2x + 4$

$$H. y = \frac{5 + x + x^3 + x^4}{x+1}$$



VA: $x = -1$
 EBA: $y = x^3 + 1$
 y-int: $(0, 5)$